

University of Bahrain

Department of Physics

**PHYCS425: Computational Physics**

**Fall 2021**

**Constrained motion on surfaces**

**Instructor**

Dr. Jawad Mohamed Taher Mohamed Alsaei

**Authors**

|  |  |
| --- | --- |
| Ali Mirza Isa | 20186917 |
| Asif Bin Ayub | 20191251 |
| Kumail Abdulaziz Radhi | 20196080 |

**Contents**

|  |  |
| --- | --- |
| 1. **Introduction** | 2 |
| 1. **Theory**   2.1- The Runge-Kutta method  2.2- | 3 |
| 1. **Part I: Constrained motion in 1D** |  |
| 1. **Part II: Constrained motion in 2D** |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

1. **Introduction**

Imagine you’re on a roller coaster. You’re plunging downwards from a ramp at dizzying speeds. You’re terrified, but not afraid for your life[[1]](#footnote-1). You know that no matter what happens, you’ll always remain on the track. This is an example of constrained motion, and it is virtually everywhere. An athlete skiing down an icy mountain, a car driving down a road[[2]](#footnote-2) are instances of constrained motion on surfaces.

There are other forms of constrained motion, like that of a fluid moving in a pipe. But our focus in this project is only on the former. These types of constraints fall under the category of “holonomic constraints”. For each one of these constraints, the system loses a degree of freedom.

If one wants to investigate such motion analytically, Newton’s laws are not really helpful. One usually resorts to Lagrangian or Hamiltonian mechanics to get any sort of insight from these systems. Even then, getting exact solutions is cumbersome, and when damping is added, it becomes exponentially more difficult to obtain. This is where computer simulations come in handy. They allow us to study the properties of motion without knowing the most general form of the solution.

In this project, we will explore constrained motion on various surfaces in both 1D & 2D. In part I, we’ll investigate the motion on 2 curves: a parabola (), and an unknown curve , which we have to identify. We also identify the equilibrium points for both. We study the periodic motion in detail, exploring the effect of various parameters like the dimension of the curves & initial displacement. We then add damping and driving forces to the systems and investigate the possibility of chaotic motion.

In part II, we have a spherical surface. We investigate the initial conditions that lead various paths – including closed paths and Lissajous-like figures. Here too, we investigate if chaotic motion is possible. In addition, we study the effects of a homogeneous damping force.

In all parts, we used 4th order RK4 method to simulate the systems. The conservation of energy and angular momentum were checked wherever relevant.

Buckle up, because you don’t want to fall off this roller coaster!

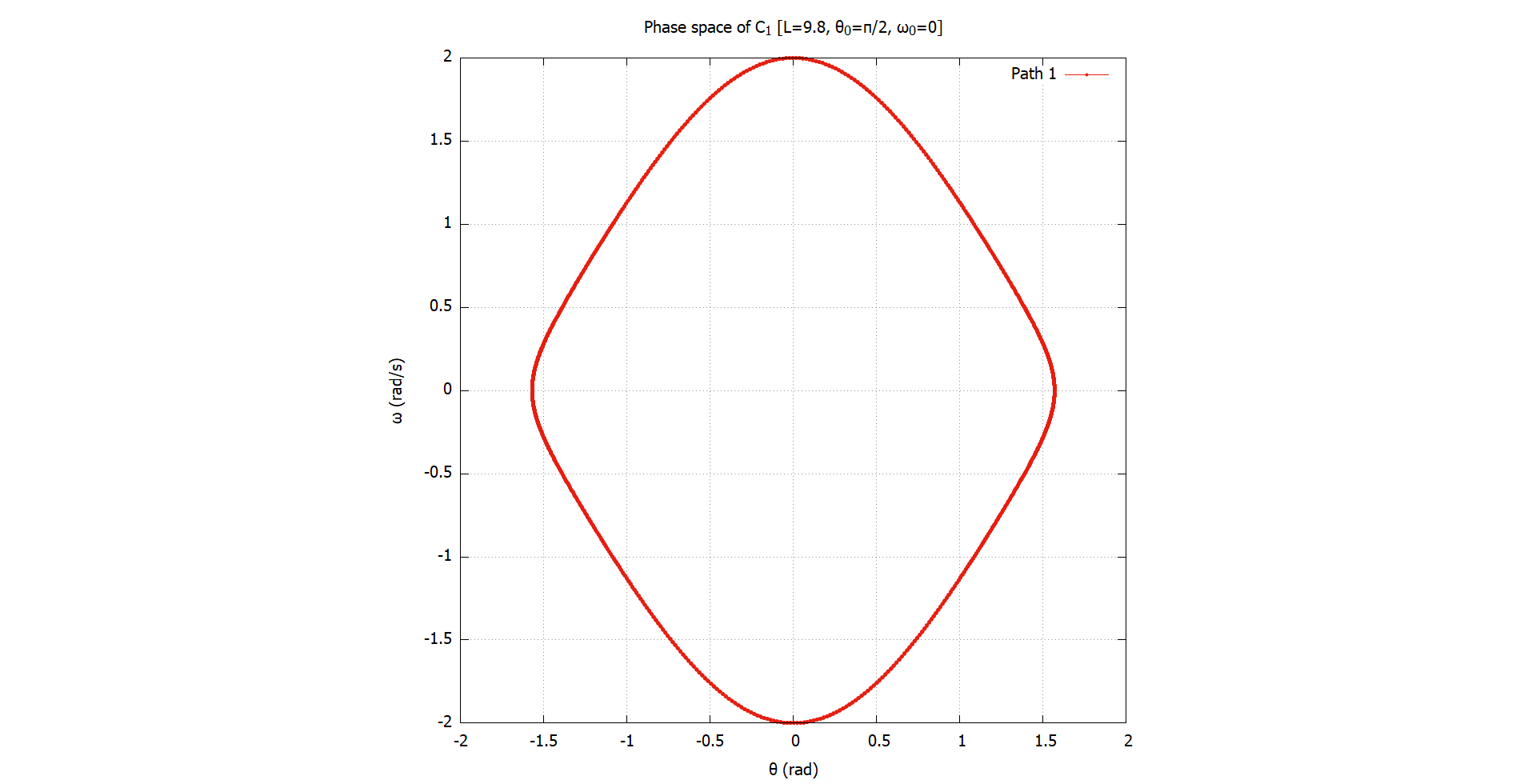
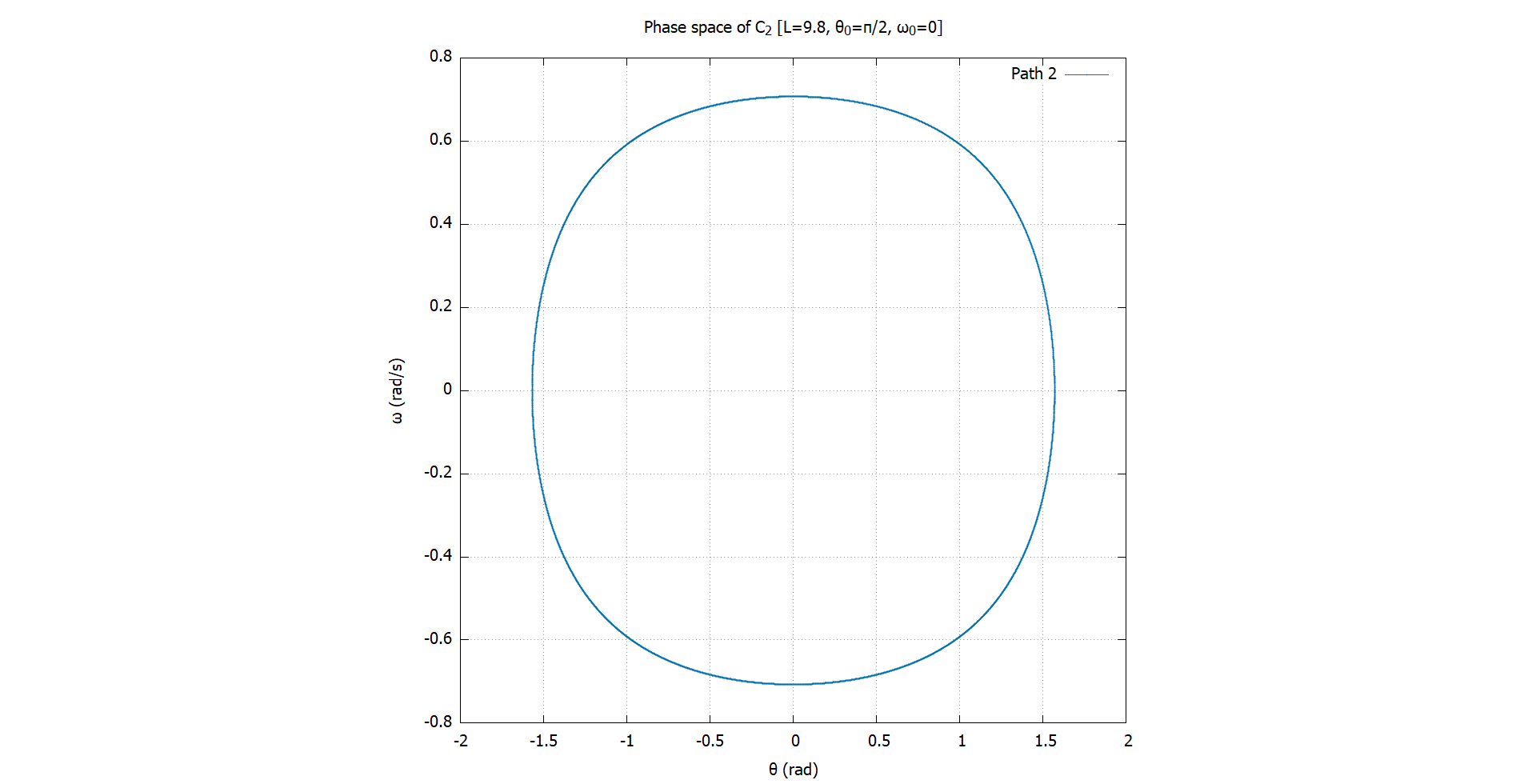
1. **Theory**

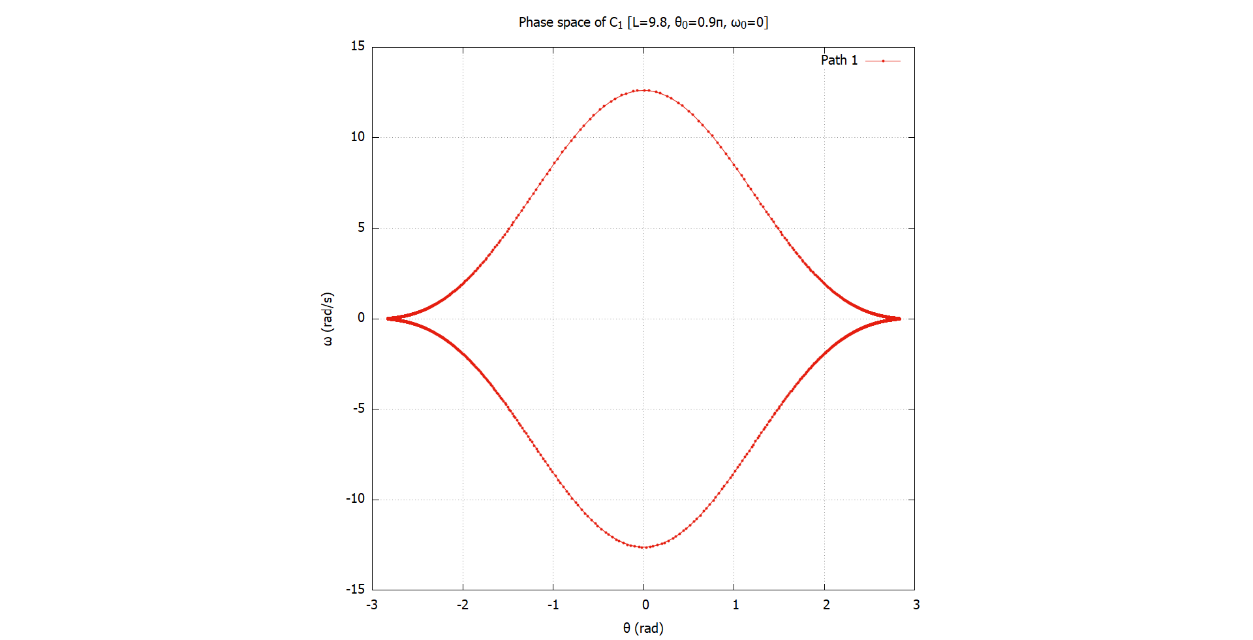
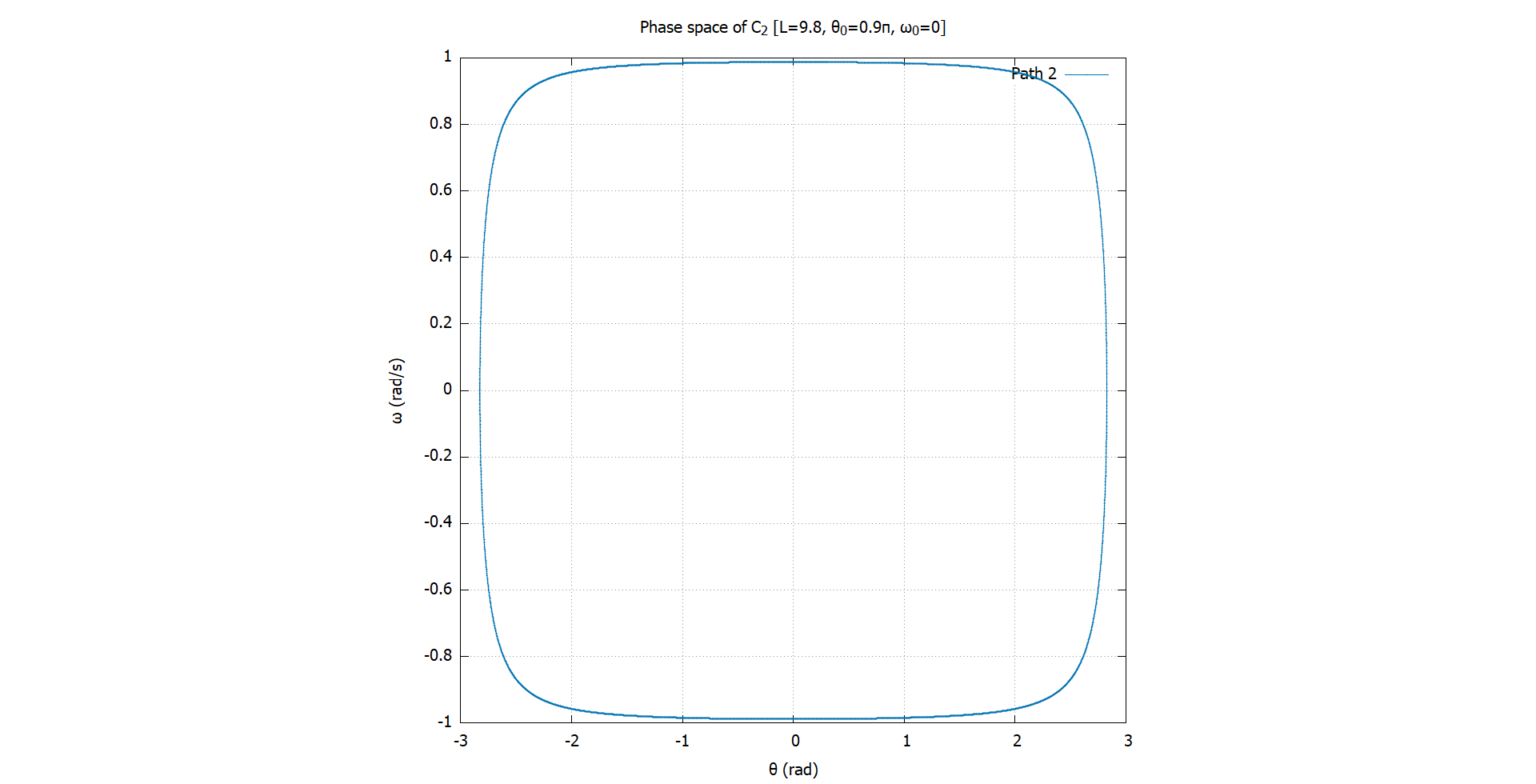
**2.1- The Runge-Kutta Method**

Since we used 4th order Runge-Kutta method (or RK4) throughout this project, it is worthwhile to go over some aspects of this method. But first, some historical context. Back in 1895, Carl Runge thought he could outsmart Euler by coming up with a numer more accurate than Euler. Euler’s method was to approximate solutions to initial value problems by . Euler’s method was

1. **Part I: Constrained motion in 1D**

**3.1- Oscillatory motion & equilibrium points**

 Trying out different initial conditions, we could confirm that a particle undergoes oscillatory motion on both curves . This corresponds to closed paths in the phase space. Shown below are the phase space portraits corresponding to the different initial conditions for both curves.



Closed paths in the phase space correspond to oscillatory motion. This is replicated for all initial conditions in the allowed range .

Classically, equilibrium points correspond to the points where the potential energy of the system is either a local minima or maxima. This corresponds to the points where the net force/torque acting on an object is 0. For determining equilibrium points, we checked at what instances of motion the angular acceleration ( or der\_omega in the code) is close to 0, up to some tolerance.

We found that both curves have one equilibrium point each, corresponding to . For the parabola (), this corresponds to its vertex, the point at which the gravitational potential is minimum. And as we will find out later on in this report, the equilibrium point for is also where gravitational potential is minimum.

Knowing that the potential energy at equilibrium points is minimum, we can conclude that these points are points of *stable equilibrium*. This will be further justified when we look at the phase portraits of damped motion.

**3.2- Dependence of oscillation period () on the oscillation amplitude ()**

1. Assuming the roller coaster was well maintained [↑](#footnote-ref-1)
2. Unless the car is in a rally race [↑](#footnote-ref-2)